Games@Carmel, May 14-17, 2018, Update

A workshop in Combinatorial Game Theory, organized by Urban Larsson, the Game Theory group at the Technion - Israel Institute of Technology, Haifa, Israel

This is a workshop in Combinatorial Game Theory (CGT), a field initiated by authors such as Bouton, Wythoff, Sprague, Grundy, Milnor, Berlekamp, Conway, Guy, Smith, Norton, and Fraenkel.

Time plan: Monday 14 May to Thursday 17 May 2018. The workshop will take place in the Bloomfield building and the Library of the Cooper Building. We booked the Library for the full workshop, so some of the time we have two rooms.

Monday 9 -11.30 am, 527 Bloomfield, 2pm to 5pm Library Cooper,

Tuesday 9.30 am to 5 pm, Library, Cooper (Lunch noon-1.30pm),

Wednesday 9.30 to 5pm, 526 Bloomfield (Lunch noon-1.30pm),

Thursday 9.30 to 5pm, 153 Bloomfield (Lunch noon-1.30pm).

If you want to join the workshop (partial or full participation is possible and there is no registration fee), you may send an email to email: urban031@gmail.com.

Monday 14 May

Room 527, Bloomfield building, IEM, Technion - Israel Institute of Technology

9.00 Carlos P. dos Santos, University of Lisbon

Extended Combinatorial Game Theory. This talk is moved to Wednesday morning. See below.

9.30 Aviezri S. Fraenkel, Weizmann Institute of Science

Cyclic games

Abstract:

CGT normally deals with games whose game-digraphs G are finite and acyclic, so always one player wins, the other loses. What happens if G is cyclic or infinite? If G is a loop, then clearly its two labels are D (Draw). What happens if a loop or cyclic digraph is attached to a vertex of an acyclic digraph? How does it affect the P and N values? Can we define a GSG (Generalized Sprague Grundy) function so as to accommodate sums? We shall give full answers to these and related questions in a leisurely, gentle, informally manner.

10.00 Gal Cohensius, Technion

Cumulative Subtraction Games

Abstract

We study a variant of classical Subtraction Games, called Cumulative Subtraction (CS), in which two players alternate in moving, and get points for taking pebbles out of a joint pile. We present the optimal play (pure subgame perfect equilibrium) in the case of two possible actions. In addition we prove that when the pile is large enough, the maximal action is optimal. This is joint work with Urban Larsson, Reshef Meir and David Wahlstedt.

Wednesday 16 May

526 Bloomfield building, IEM, Technion - Israel Institute of Technology

9.30 Urban Larsson, Technion.

Properties of Cumulative Games

Abstract

We study variations of the classical zero-sum subtraction games, called cumulative games; we generalize the games presented by Gal Cohensius in the previous talk. This new class includes all n-players general-sum extensive form games. As a main result, we demonstrate when the value in pure subgame perfect equilibrium equals a certain play-value plus the current cumulation. In this case, the game value has an 'efficient description' via an outcome function. We discuss how to adapt the cumulation games to disjunctive sum, partial order game comparison, and other common concepts from Combinatorial Game Theory. This is joint work with Reshef Meir.

10.00 Mike Fisher, West Chester University

Beatty Subtraction Games

Abstract:

A Beatty sequence is a sequence of integers formed by taking the floor of the positive integral multiples of a positive irrational number α. The complementary sequence is formed in a similar manner using β, where β satisfies the equation $\frac{1}{\alpha} + \frac{1}{\beta} = 1$. For a given α, we investigate the partizan subtraction game with left and right subtraction sets given by $(1, \alpha) = \{1\} \cup \{\lfloor n\alpha \rfloor \mid n \in \mathbb{Z}^+\}$ and $(1, \beta)$, respectively. We analyze this family of games using the Atomic Weight Calculus.

Octal games are impartial games that involve removing tokens from heaps of tokens. These types of games are interesting in that they can be uniquely described using an octal code. Historically, research efforts have focused almost exclusively on octal games with finite codes. We consider octal games based on infinite octal codes where the heap sizes corresponding to elements of a Beatty α sequence are played according to some fixed removal rule and the heap sizes corresponding to elements of a Beatty β sequence are played according to some other fixed removal rule. Interesting periodicity seems to occur in most cases.

10.30 Reshef Meir, Technion

Combinatorial Bidding Games

Abstract:

Bidding games are extensive form games, where in each turn players bid in order to determine who will play next. Zero-sum bidding games like Bidding Tic-Tac-Toe (also known as Richman games) have been extensively studied [Lazarus et al., Develin and Payne].

We extend the theory of bidding games to general-sum two player games, showing the existence of pure subgame-perfect Nash equilibria (PSPE), and studying their properties. In particular, we show that the set of all PSPEs forms a semilattice, whose bottom point is unique.

Our main result shows that if the underlying game has the form of a binary tree (only two actions available to the players in each node), then the Bottom PSPE is monotone in the budget, Pareto-efficient, and fair.

In addition, we discuss applications of bidding games to combinatorial bargaining, and provide a polynomial-time algorithm to compute the Bottom PSPE.

To appear in GEB. Joint work with Gil Kalai and Moshe Tennenholtz.

11.00 Carlos P. dos Santos, University of Lisbon

Extended Combinatorial Game Theory

Abstract:

About disjunctive short sum, John Conway wrote the following: The compound game ends as soon as any one of the component games has ended (ONAG, 1976). In fact, this rule is directly related to the idea of terminal moves: moves that imply the end of the play, even if the players still have available options. The end of a component, the capture of a piece (e.g., chess king), a specific goal, are examples of terminal rules related to terminal moves.

The game values of terminal moves should be infinite. It is necessary to consider the extended game line in exactly the same way we can consider the extended real number line (with infinities). But an understanding of the algebra is mandatory (game comparison, disjunctive sum, inverses). Success in this task will allow better research on disjunctive short sum of classic Combinatorial Game Theory (e.g., Atari Go), and better approaches for scoring combinatorial rulesets (e.g. Dots and Boxes). We will discuss and

exemplify some problems behind this idea. This is joint and ongoing work with Urban Larsson and Richard J. Nowakowski.